Comments on “Dynamical properties of hybrid automata”, IEEE Trans. on Automatic Control, vol.48, pp.2-14, 2003

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Thème 4 — Simulation et optimisation
de systèmes complexes
Projet Bipop

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Abstract: The above-mentioned paper contains a general analysis of hybrid dynamical systems. Well-posedness and stability results are presented. In this note we point out some limitations of these results which are somewhat hidden by a quite unfortunate choice of the illustrating example. Actually it happens that so-called complementarity mechanical systems are, to a large extent, outside the scope of the developed theory, contrary to what the example might let one believe.

Key-words: Hybrid systems, complementarity systems, unilateral constraints, well-posedness.

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Commentaires sur l'article "Dynamical properties of hybrid automata", IEEE Trans. on Automatic Control, vol.48, pp.2-14, 2003

Résumé : Cette note contient quelques commentaires sur un article consacré à la dynamique des systèmes hybrides. Nous montrons les limites de l'analyse faite dans cet article, qui sont masquées par un choix malheureux de l'exemple illustratif repris tout au long de l'article. En fait les systèmes dits de complémentarité ne rentrent pas, en général, dans le cadre de la théorie développée dans ce papier.

Mots-clés : Systèmes hybrides, Systèmes de complémentarité, contraintes unilatérales, Systèmes bien posés.

1 Introduction

The above-mentioned paper deals with several aspects of hybrid dynamical systems like well-posedness (existence of solutions, uniqueness of solutions, continuous dependence on initial conditions), and stability. This is certainly a nice and important contribution to hybrid dynamical systems analysis, and we would like to point out some of its limitations which most readers might not suspect due to the unfortunate choice of the illustrating example. In all what follows we shall deal with mechanical systems, whose vector of generalised coordinates is denoted as $x \in \mathcal{C}$, $\mathcal{C} \subseteq I\!R^n$ is the configuration space. Let us make the following comments about the above-mentioned paper:

1 The theoretical results are illustrated by the so-called rocking block. However the dynamical model of the rocking block that is studied (section II.E) is not acceptable from a mechanical point of view, because:

  1a The used restitution law is a simplistic rule, lacking of physical foundation,

  1b The dynamical model is incomplete.

2 A much better model consists of embedding the rocking block into so-called *complementarity dynamical systems*, which form a specific class of hybrid dynamical systems, but can also be viewed as differential inclusions, see [5, 6]. This allows one, in particular, to solve the problem raised at the end of sections II.E and IV.A about fixed points of the rocking block.

3 Many mechanical systems subject to unilateral constraints, do not possess the continuous dependence on initial conditions property [1, 7], which is crucial for the extension of the LaSalle's invariance lemma. In fact, it happens that the conclusions of Lemma III.3 do not hold for many complementarity mechanical systems subject to *multiple unilateral constraints* [2, 3]. In particular, the rocking block typically shows such discontinuities, for a large range of its parameters (mass, dimensions). On the other hand there exist complementarity mechanical systems that do not satisfy Assumption 4) of Theorem III.2 but whose solutions are continuously dependent on initial conditions.

4 The reachable sets and $\omega-$limit sets of mechanical complementarity systems may be quite peculiar as we shall see with simple examples. Hence possibly $\mathrm{Reach}_H \subset \mathrm{Init}$, violating Assumption II.2 on which Lemma IV.1 relies.

5 Finally the well-posedness of mechanical complementarity systems (existence, uniqueness of solutions) has been long studied in the Applied Mathematics literature. The most recent and important contributions are in [1, 16]. Analycity of the data is crucial to get uniqueness of solutions [1]. The global Lipschitz continuity of vector fields as required in Assumption II.1 is not satisfied for most Lagrangian systems. Also, the exact nature of solutions as stated in Definitions II.2, II.3 and Theorem III.1 is not very clear in this setting.

General model and well-posedness: These points raise essential features of mechanical complementarity systems (MCS), which to our opinion justify the fact that MCS have to be considered apart from the theory developed in the above-mentioned paper. In the following we will support our claims by simple examples. First of all, let us state that the rocking block system, with instantaneous velocity jumps at impacts, belongs to the following class of non-smooth dynamical systems, which has been introduced by Moreau [8, 9, 10]:

$$\left\{\begin{array}{l} M(x)\ddot{x}+F(x,\dot{x},t)=\nabla h(x)\lambda \\ \\ 0\leq h(x)\perp\lambda\geq 0 \\ \\ \dot{x}(t_k^+)=\arg\min_{z\in V(x(t_k))}\frac{1}{2}Z^TM(q(t_k))Z \end{array}\right. \tag{1}$$

where:

- $M(x)$ is the positive definite inertia matrix, $F(x,\dot{x},t)$ contains Coriolis, centrifugal, conservative, and external forces/torques, $h(x)\in I\!R^m$, $\lambda\in I\!R^m$ is a vector of Lagrange multipliers, also called a slack variable in nonlinear programming, $\nabla.$ is the Euclidean gradient,
- the second line is a *complementarity relation* between $h(x)$ and λ (i.e. a particular *contact model*, which is missing in the above-mentioned paper),
- the third line is an impact rule, $Z=z-\dot{x}(t_k^-)$, $V(x(t))$ is the tangent cone to the admissible domain $\Phi=\{(x,\dot{x})\mid h(x)\geq 0\}$ [10, 11], $h(\cdot)$ is such that $V(x)\neq\emptyset$ for all $x\in\Phi$,
- t_k generically denotes the impact times, at which the velocity $\dot{x}(\cdot)$ undergoes a jump calculated from the impact rule, $\dot{x}(t_k^+)=\lim_{t\to t_k,t>t_k}\dot{x}(t)$, $\dot{x}(t_k^-)=\lim_{t\to t_k,t<t_k}\dot{x}(t)$.

One notices from (1) that MCS possess in fact a unique vector field, and that their dynamics switches between lower-dimensional subspaces. This places MCS in a slightly different perspective from many other types of hybrid systems (though they can be embedded into Definition II.1).

Remark 1 *The complementarity relations cannot be dispensed with. They monitor the detachment conditions during phases of motion with $h_i(x)=0$ for some $1\leq i\leq m$. On such phases, the multiplier is the solution of a linear complementarity problem [8, 5, 12, 13]. They are also fundamental for the characterisation of equilibrium points, as shown below.*

The rocking block system can be recast into (1), where $y_A(x)$ and $y_B(x)$ may be chosen as the unilateral constraints $h(x)^T=(y_A(x),y_B(x))$, as shown on figure 1. In order to simplify the analysis one may further assume that the contact points do not slide on the boundary $\partial\Phi$, despite the constraint is frictionless. This assumption, though not very realistic from a Mechanics point of view, has little influence on the material discussed in this note. The next theorem is a compilation of proposition 32, problem $\mathcal{P}$, theorems 8, 10 and corollary 9 of [1]. The results in [1] constitute the most advanced results about frictionless MCS to date.

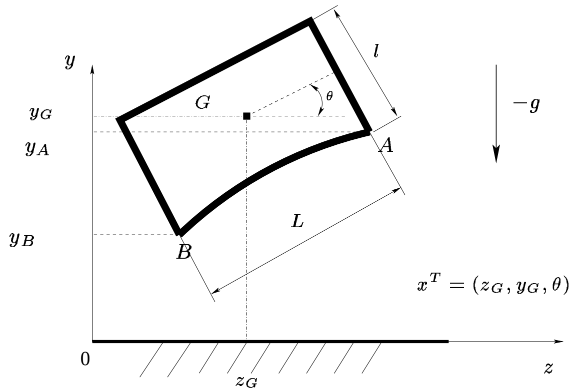

Figure 1: The planar rocking block.

Theorem 1 *[1] Assume that the configuration manifold $\mathcal{C}$, the functions $h_i(x)$, $1 \le i \le m$, and the mapping $F(x, \dot{x}, t)$ are analytic, the functions $h_i(x)$ are functionaly independent, and that $||F(x, v, t)||_x \le l(t)(1 + d(x, x(0)) + ||v||_x)$ with $l(t) \in L^1_{loc}(\mathbb{R}; \mathbb{R}^+)$, $d(x, x(0))$ is the Riemannian distance between x and the initial position $x(0)$, and $||\cdot||_x$ is the kinetic norm. Then solutions of (1) exist on $\mathbb{R}^+$ and are unique with $x(\cdot)$ absolutely continuous, whereas the velocity $\dot{x}(\cdot)$ is right continuous of local bounded variation. Moreover the acceleration is a measure $dv = \{\ddot{x}\}dt + d\mu_a$, where $\{\ddot{x}\}$ is a Lebesgue integrable function, and $d\mu_a$ is an atomic measure with a countable set of atoms on any compact time interval (atoms coincide with impact times).* ■

In other words the model in (1) is complete and can be integrated on $\mathbb{R}^+$. We note that the results in [1] permit more general impact rules than in (1), see [1, Equ.(16)]. Theorem 1 appears to be much more powerful than its counterpart Theorem III.1, since it applies to non-autonomous MCS and provides one with accurate and usable informations on the nature of the solutions, which has considerable importance in view of subsequent analysis and control, see below. Definitions II.2 and II.3 do not contain the solutions in the sense of theorem 1, because the event times are orderable, the state $x(\cdot)$ may not be of local bounded variation, and solutions should be differentiable on the intervals $I_i = [\tau_i, \tau_i']$ or $I_N = [\tau_N, \tau_N')$ when $N < +\infty$: this is clearly not satisfied by the solutions as in theorem 1 ([1]). The fact that dv possesses the above decomposition, has consequences on the Lyapunov

[1]The function $x : t \mapsto (t-1)\sin(\frac{\pi}{t-1})$ with $x(1) = 0$ is continuous on $[0, 1]$, it is infinitely differentiable on $[0, 1)$, but is it not of bounded variation on $[0, 1]$ (take $t_k - 1 = \frac{2}{2k+1}$, $k \ge 0$ to check that its variation

stability analysis that can be led by studying the density of the Lyapunov function derivative with respect to the positive measure $d\mu = dt + d\mu_a$ [15]. It is noteworthy that most of the well-posedness results obtained so far for MCS, do not rely on any hybrid systems point of view, but rather on considering MCS as (non-standard) differential inclusions and making use of convex analysis tools [2, 7, 9, 10, 16], an exception being [3]. The addition of Coulomb friction to (1) further complicates the analysis [16] and places the simplest MCS far outside the scope of the above-mentioned paper. Therefore Definitions II.2 and II.3, and Theorem III.1 are inadequate to study MCS. It is noteworthy that the violation of Assumption II.1 is not the main problem but rather a technical point.

Zeno hybrid systems: Whether or not systems whose solutions undergo infinitely many "events" in finite time are included in the analysis of the above-mentioned paper is not very clear. Several sentences ("Zeno hybrid automata will not be studied further in this paper" – end of section III.A –, "The conclusion of this example could also have been derived using the property of Zeno executions" – end of section IV.A –, "some open problems...: Zeno hybrid systems," – section V –) leave the reader with a fuzzy impression. MCS usually are Zeno, and the measure $d\mu_a$ (see theorem 1) can take a complex form because the set of discontinuity times of a function of bounded variation may not be orderable.

The impact rule: Equivalently one may define the restitution law (corresponding to the reset map $R(e,x)$ in the above-mentioned paper) as

$$\nabla h_i(x(t_k))^T \dot{x}(t_k^+) = -e_i \nabla h_i(x(t_k))^T \dot{x}(t_k^-) \tag{2}$$

for all constraint surfaces $\{x \mid h_i(x) = 0\}$ which are active at t_k, and taking $e_i = 0$. This combined with the algebraic Lagrangian dynamics at t_k, allows one to retrieve the impact rule in (1). Thus if $m = 1$ this impact rule reduces to the standard inelastic Newton's law. A non-zero restitution can easily be introduced. The impact rule in (1) is not the only possible one [12]. However a choice has to be made for analysis and control purposes. This one has limitations, on which we shall not extend in this note. It is however definitely much better, from a Mechanical point of view, than the rule presented in the above-mentioned paper Section II.E, which is hardly acceptable. For instance, if the block collides the ground with corner B while A is airborne, one applies a Newton's restitution to the normal velocity at B, with coefficient $e_n \in [0,1]$. In case of a collision between two frictionless bodies, such a modelling choice can be refined but is widely accepted as a good model [18]. This will necessarily result in $\dot{\theta}(t_k^+) = R(m,l,L,\dot{y}_B(t_k^-),e_n)\dot{\theta}(t_k^-)$, as can be calculated from [12, Equ.(6.83)] by choosing $\lambda_1 = 0$. There is no reason that $R(m,l,L,\dot{y}_B(t_k^-),e_n) \in [0,1]$ in order to guarantee kinetic energy loss [12, Chapter 6]. Conversely stating that $\dot{\theta}(t_k^+) = e\dot{\theta}(t_k^-)$ may result in an impact law not satisfying [1, Proposition 5]. Clearly the restitution law in

diverges since the series $\sum \frac{1}{n}$ diverges.). Such a function apparently satisfies the requirements in Definitions II.2 and II.3. In other words, the executions as they are defined in the above-mentioned paper do not imply that $x(\cdot)$ is of local bounded variation. Finally functions of bounded variation possess a countable set of discontinuities, but this set may not be orderable.

(2) satisfies Assumption 3) of Theorem III.2, whereas the conditions of Lemma III.1 read as $\dot{x}(t_k^+) \in V(x(t_k))$ and $V(x(t_k)) \neq \emptyset$. It satisfies the constitutive hypothesis on which theorem 1 above relies [1, §3.3, Equ.(53)], while the other restitution law generally does not and should be disregarded despite its apparent simplicity. In summary, in order to apply theorem 1, one has to start from an impact rule similar to the one in (1), which definitely does not lead to a constant coefficient $e \in [0,1]$ and $\dot{\theta}(t_k^+) = e\dot{\theta}(t_k^-)$.

Characterisation of equilibrium points:

Lemma 1 *The fixed points x^* of the MCS in (1) are solutions of the generalised equation*

$$\left\{ \begin{array}{l} F(x^*, 0, t) = \nabla h(x^*)\lambda \\ \\ 0 \le h(x^*) \perp \lambda \ge 0 \end{array} \right. \tag{3}$$

■

where it is taken into account that $\dot{x}(t_k^+) = 0$ if $\dot{x}(t_k^-) = 0$, because $0 \in V(x)$ for all x (similar to Assumption 2) of Definition II.6). In particular, solutions of (3) contain not only the two (unstable) equilibria given in section II.E of the above-mentioned paper, but the (stable) equilibrium that corresponds to the block at rest on the boundary $\partial\Phi$ (thus (3) is in this case the *static equilibrium* of the mass on the ground, subject to gravity and ground reaction λ). This is important since this is indeed the only fixed point of interest for stability. The complementarity relations in (1) are necessary to take this stable equilibrium point into account. To better fix the ideas, consider the simplest MCS, i.e. a one degree-of-freedom bouncing ball. If one doesn't take complementarity into account, the dynamics is given by $\left\{ \begin{array}{l} \ddot{x} = -g \\ \dot{x}(t_k^+) = -e\dot{x}(t_k^-) \end{array} \right.$, which obviously possesses no fixed point! Completing the model as $\left\{ \begin{array}{l} \ddot{x} = -g + \lambda \\ 0 \le x \perp \lambda \ge 0 \\ \dot{x}(t_k^+) = -e\dot{x}(t_k^-) \end{array} \right.$ allows one to state that $(x, \dot{x}) = (0,0)$ is the (static) equilibrium. Compare (3) with the conditions in Definition II.6: condition 1) of Definition II.6 does not take into account the fact that MCS may live on lower dimensional subspaces, and that the vector field is modified by the multiplier λ. So the conclusion that can be found at the end of section II.E: "$(0,0)$ is not an equilibrium, since it violates the first condition of Definition II.6", no longer holds if the complementarity conditions are included in the model. Actually it could even be said that condition 1) of Definition II.6 is correct, if one accepts that the vector field $f(q,x)$ is modified with the suitable multiplier when the system evolves on the boundary $\partial\Phi$.

Continuous dependence on initial conditions: Let us now study a quite simple case of a MCS, namely a particle that bounces in a corner, see figure 2. The kinetic angle [12, §6.2] between the two constraint boundaries (equal in this particular case to the Euclidean angle) is denoted as θ_{kin}. If $\theta_{kin} \in (0, \frac{\pi}{2})$ and the inelastic impact law in (1) is used, or if $\theta_{kin} = \frac{\pi}{2}$, then trajectories of (1) depend continuously on the initial data [7, 1]. When either $\theta_{kin} \in (\frac{\pi}{2}, \pi)$ or the restitution on each surface is not zero, then solutions of (1) are discontinuous in the initial data (more accurately, the mapping which associates the state $(x(t), \dot{x}(t))$ at time t to the initial state $(x(0), \dot{x}(0^-))$ at $t = 0$ – sometimes called the *flow with collisions* [12]– is discontinuous in the initial condition after the first impact). This is easily visualised by choosing for instance $\theta_{kin} = \frac{2\pi}{3}$, $e_i = 1$, and initialising the system on both sides of the bissector (OO') with a vertical velocity. It is crucial to note that this is not due to the particular choice of the impact rule in (1) when the particle hits the corner at O, but is intrinsic to the system (as soon as one accepts that Newton's law is a correct model for systems impacting a single smooth constraint). Exactly the same phenomenon occurs with the rocking block, depending on its dimensions and whether corner A hits before corner B or the converse (the kinetic angle is equal to $\frac{\pi}{2}$ when $l = \frac{\sqrt{3}}{2}L$ with an inertia momentum $I = \frac{m}{12}(l^2 + L^2)$ [12, p.317]). Discontinuous dependence on initial conditions is therefore a basic characteristic of most MCS [1, §7], see also [3] [4, Example 3.3] for other examples. Consequently the conclusions of Lemma III.3 do not apply to most of MCS with $m \geq 2$. Moreover some MCS with $m \geq 2$ do not satisfy Assumption 4) of Theorem III.2, but they do enjoy the continuous dependence property [7]. Theorem III.2 and Lemma III.3 are inadequate to study MCS. In this setting it is also worth reading [17].

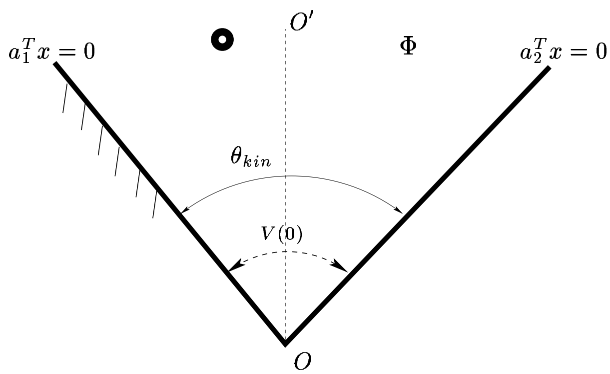

Figure 2: A particle in an angle.

Reachable sets: Let us continue with the particle example and the impact rule in (1), with the origin taken at the corner O. Let us assume that $\theta_{kin} \in (0, \frac{\pi}{2})$. Then all trajectories that collide the corner O at time t_k, come at rest at O, i.e. $x(t_k) = 0$ and $\dot{x}(t_k^+) = 0$. Let us also suppose that there is an external force $u(t)$ acting on the particle playing the role of an admissible bounded input (such that the conditions of theorem 1 are fulfilled). Then all states $(0, \dot{x}_0)$ with $\dot{x}_0 \neq 0$, $\dot{x}_0 \in V(0)$, are unreachable from any state with $x \in \Phi \setminus \partial\Phi$. Indeed the velocity cannot instantaneously jump to a non-zero value with bounded functions

$u(\cdot)$. However from a mathematical point of view the system can be initialised with such states. Finally, since velocities are right-continuous functions, the states with $x \in \partial\Phi$ and $\dot{x} \in -V(x)$ cannot be reached neither. In fact only the origin $(0,0)$ (corner O on figure 2) and states with $x \in \partial\Phi$ and $a_i^T \dot{x} = 0$ can be reached from any state with $x \in \Phi \setminus \partial\Phi$. Consequently Assumption II.2 of the above-mentioned paper is not satisfied, and one sees that MCS possess quite specific reachable subspaces.

$\omega-$limit sets: Let us consider again the one degree-of-freedom bouncing-ball dynamics. From theorem 1 the velocity is right-continuous of bounded variation. The $\omega-$limit set of a trajectory with $x(0) > 0$ is equal to $\{(0,0)\}$ if $e = 0$. Now if $e = 1$, then the $\omega-$limit set is $[0, x_M] \times (-\dot{x}_M, \dot{x}_M]$ which is not compact, contradicting Assumption II.2 and consequently Lemma IV.1. This shows that exact nature of trajectories is of utmost importance for subsequent analysis.

Conclusion: Mechanical complementarity systems are to a large extent outside the scope of the theoretical results developed in the above-mentioned paper, and the choice of the rocking block model as an illustrating example is quite unfortunate. Simple frictionless examples are considered to support the claims of this note. It is noteworthy that the mere addition of Coulomb friction (that is encompassed by complementarity) complicates the dynamics in a such a way [14, 16] that it seems hopeless to recover the results by embedding MCS into a more general framework. In fact, MCS possess a very rich dynamics, and it is our opinion that trying to embed MCS into a larger class of hybrid dynamical systems is at best useless. Specifying which subclass of complementarity systems (not necessarily mechanical) can be analysed with the tools proposed in the above-mentioned paper, might be interesting.

References

[1] P. Ballard, 2000 "The dynamics of discrete mechanical systems with perfect unilateral constraints", Archives for Rational Mechanics and Analysis, vol.154, no 3, pp.199-274.

[2] M. Schatzman, 1978 "A class of nonlinear differential equations of second order in time", Nonlinear Analysis, Theory, Methods and Applications, vol.2, no 3, pp.355-373.

[3] W.P.M.H. Heemels, J.M. Schumacher, S. Weiland, 2000 "Linear complementarity systems", SIAM J. Applied Math., vol.60, no 4, pp.1234-1269.

[4] M.D.P. Monteiro-Marques, 1993 *Differential Inclusions in Nonsmooth Mechanical Problems: Shocks and Dry Friction*, Birkhauser, Boston, PNLDE 9.

[5] B. Brogliato, 2003 "Some perspectives on the control and analysis of complementarity dynamical systems", IEEE Transactions on Automatic Control, vol.48, no 6.

[6] W.P.M.H. Heemels, B. Brogliato, 2003 “The complementarity class of hybrid dynamical systems”, European Journal of Control, in press.

[7] L. Paoli, 2002 “A numerical scheme for impact problems with inelastic shocks: a convergence result in the multi-constraint case”, Proc. Int. Conference Nonsmooth/Nonconvex Mechanics with Applications in Engineering, Editions Ziti, Thessaloniki, Greece, pp.269-274.

[8] J.J. Moreau, 1963 "Les liaisons unilatérales et le principe de Gauss", C.R. Acad. Sc., Paris, 256, pp.871-874.

[9] J.J. Moreau, 1983 "Liaisons unilatérales sans frottement et chocs inélastiques", C.R. Acad. Sc. Paris, t.296, série II, pp.1473-1476.

[10] J.J. Moreau, 1988 "Unilateral contact and dry friction in finite freedom dynamics", CISM Courses and Lectures no 302, International Centre for Mechanical Sciences, J.J. Moreau and P.D. Panagiotopoulos (Eds.), Springer-Verlag.

[11] J.B. Hiriart-Urruty, C. Lemaréchal, 2001 *Fundamentals of Convex Analysis*, Springer Grundlehren Text Edition, Berlin.

[12] B. Brogliato, 1999 *Nonsmooth Mechanics*, Springer CCES, 2nd edition, London. Erratum and addenda available at http://www.inrialpes.fr/bip/people/brogliato/brogli.html

[13] B. Brogliato, A.A. ten Dam, L. Paoli, F. Génot, M. Abadie, 2002 “Numerical simulation of finite dimensional multibody nonsmooth mechanical systems”, ASME Applied Mechanics Reviews, vol.55, no 2, pp.107-150.

[14] F. Génot, B. Brogliato, 1999 “New results on Painlevé paradoxes”, European Journal of Mechanics A/Solids, vol.18, no 4, pp.653-678, July-August.

[15] B. Brogliato, 2001 "Absolute stability and the Lagrange-Dirichlet theorem with monotone multivalued mappings", 40th IEEE Conference on Decision and Control, Orlando, USA, December 4-7. INRIA report RR-4794, http://www.inria.fr/rrrt/index.en.html

[16] D.E. Stewart, 2000 "Rigid-body dynamics with friction and impacts", SIAM Review, vol.42, no 1, pp.3-39.

[17] M. Schatzman, 1998 "Uniqueness and continuous dependence on data for one-dimensional impact problem", Math. Comp. Modeling, vol.28, no 4-8, pp.1-18.

[18] W.J. Stronge, 2000 *Impact Mechanics*, Cambridge University Press.

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